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Phase I of II Flow Extension for Selected Gages in the Western U.S.

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# PHASE I of II

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Title: Row Extension for Selected Gages in the Western U.S.

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Streamflow Record Extension in the San Juan National Forest, Colorado Douglas B. Moog and Peter J. Whiting, Case Western Reserve University June 15, 2001

#### Introduction

This text reports on the extension of stream gage records in the San Juan National Forest in Colorado, such that each extended record covered at least water years 1969-98. Extensions were performed by relating each existing record to another gage in the Forest covering this period. This report discusses the selection of these pairs of gages, data preparation, and the extension method.

All flows considered herein are daily flows as reported by the USGS NWIS-W, and all extended records are presented as sets of daily flows. The word *extension* is used to indicate not only an increase in the overall time span of a gage, but also the estimation of missing dates within that period. The gage record to be extended is called the *short* record, while the one used to extend it is known as the *base* record.

#### Records extended

The gages in the San Juan National Forest for which daily flow records were extended are listed in the Appendix. Of the 52 gages in the Forest, the 49 having USGS records were analyzed; the others were deferred pending acquisition of state data. Of those 49, five had continuous records for the 1969-1998 period and so were used as base stations to extend the other records. Of the other 44, eight were found to be unsuitable for extension using the available base records. As a result, the record at each of 36 gages was extended for a period corresponding to the extent of its base record, which in all cases covered at least water years 1969-1998.

The suitability for extension of each short record was assessed on the basis of its relationship to that base record having the greatest correlation to the short record's daily flow values during the *concurrent period*, the set of days on which both gages recorded discharges. The assessment employed both the correlation coefficient r and scatterplots of the concurrent daily flows at the two gages. Screening rules were based primarily on experience. They may be summarized as:

- 1. Reject records with a concurrent period shorter than four years, to insure suitable representation of the statistical relationship.
- 2. Reject all records with a concurrent-period correlation below 0.75.
- 3. Reject records with unusual structure in a scatterplot of the short vs. base record. This is generally a result of flow regulation, and such cases might be rehabilitated by removal of a portion of the record.
- 4. Accept all remaining records with a concurrent-period correlation above 0.85.
- 5. The suitability of the remaining records is assessed on the basis of
  - a. the overall concurrent-period correlation,
  - b. length of the concurrent period,
  - c. homoscedasticity (uniformity of variance) or unusual structure in the scatterplot,

- d. the number of flows recorded as zero, and
- e. the number of flows so low that the gage was imprecise and possibly inaccurate.

In the San Juan National Forest, one record (6) was rejected by rule 1 and four (4, 5, 18, 21) by rule 2. Record 7 was rejected by rule 3. Rule 4 passed 36 of the remaining 39 records. Of the remaining 2 records subject to judgment (rule 5), one (42) was accepted and one (13) rejected, for a total of 37 extended short records.

The causes of low correlation and the criteria of rule 5 are examined in greater detail later in this text.

## **Data Preparation**

All flows were downloaded from the USGS NWIS-W data set at <a href="http://waterdata.usgs.gov/nwis-w/US/">http://waterdata.usgs.gov/nwis-w/US/</a>. Estimated flows were accepted as given. In the cases of two gages, periods of flow were discarded owing to the presence of flow regulation. These were record 22 after water year 1970, and record 30 after water year 1940.

The only data preparation was the handling of zero flow data. In the extension process, all flow data were subject to transformations (such as the logarithm) which cannot be used with zero flows. Three of the records which were extended had zero data: 19, 46, and 47, with zeroes comprising 4.4%, 0.3%, and 2.7% of the data, respectively.

Handling of these flows followed a procedure described by Stedinger et al. (1993). Zero flows may be regarded as those with magnitudes below the detection threshold of the gage, and thus below all the measured values. The set of zero dates was assigned a set of exceedance probabilities according to the Weibull plotting position, i/(n+1), where i is the rank of the flow and n is the number of dates. The flow magnitudes for this set were taken as the lower tail of a lognormal distribution, such that the highest flow in the set equaled the lowest nonzero recorded flow, and the mean of the overall lognormal distribution equaled the mean log flow of the set formed by the union of the observations with the set of zero dates.

While the values assumed for the zero-flow dates generally matter little in application - the difference between zero and the lower detection limit being insignificant - the values used in the extension can make a difference owing to transformations. The objective of filling in these data therefore was to make them as consistent as possible with the known data while falling below the known data.

### Extension technique

Each extended record is the union of two sets of flows: (A) the known flows for that gage and (B) flows generated for dates covered by the base stream gage, but not appearing in the short record. The calculation of short-record flows from base station flows proceeds from the generation of an isomorphic, continuous, monotonic curve relating the records.

## MOVE.1 with scaled power transformations

The relationships used to extend the records from the base stations were generated using an extension of the technique described by Moog et al. (1999). In this technique, the data for each record are subjected to a scaled power transformation:

$$Q' = \begin{cases} \frac{Q^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln Q & \text{if } \lambda = 0 \end{cases}$$
 (1)

where Q is the flow rate and  $\lambda$  is an exponent chosen by a maximum likelihood criterion. Note that the logarithmic transformation is simply a special case of the more general scaled power transformation, which can linearize relationships which would be curved in log-log space. The goal of the transformation is to condition the data so that the relationship between the records is more linear and homoscedastic when a linear fit is calculated.

The MOVE 1 equation (Hirsch, 1982) is used to relate the transformed data linearly such that the estimated mean and variance for the concurrent data equal the mean and variance of the measured data. The equation takes the form

$$\hat{y}_E = m(y_C) + \frac{S(y_C)}{S(x_C)} (x_E - m(x_C))$$
(2)

where  $\mathcal{Y}$  indicates a set of estimated flows, y a set of short-record flow measurements, x a set of base record flows, subscript C the concurrent period, and subscript E the rest of the extension period. The functions m and S are the sample mean and standard deviation. In the current application, x and y were transformations of the flow rate using Eq. (1). This equation is the same as that for linear least-squares regression with the coefficient r of the last term replaced by unity. Thus the slope of the line is greater than that produced by least-squares linear regression, equal to that of a record perfectly correlated to the base record. The result is greater variance in the extended record than would be produced by least-squares linear regression.

After application of Eq. (2), the results are retransformed - that is, back to flows in cfs - by inverting Eq. (1). The retransformation may produce complex numbers for some values of  $\lambda$ ; the values are thus chosen conservatively following the guidelines of Moog et al. (1999).

## Modifications to MOVE. I for retransformation bias

For generation of streamflow records, the Moog et al. (1999) technique has one shortcoming: retransformation bias. While the mean and variance of the measured and estimated data are equal for the transformed variables, they differ after retransformation to cfs. This was not a problem in the 1999 paper, for which only the shape of the flow duration curve mattered, not the actual magnitude of flow. While there may be an argument that matching in the transformed space is preferable, hydrologists would generally prefer a formula which produces the same mean flow in cfs as that of the measured data (during the concurrent period).

For this reason, the MOVE.1 technique was modified to remove the retransformation bias. While there are a number of bias correction techniques for methods, like linear least-squares regression, which estimate the line of means, we knew of none available for MOVE.1. However, the desired modification is easily defined; the new MOVE should match the measured mean and variance in the original space (e.g., cfs) rather than the transformed space, while remaining linear in transformed space.

The two constraints (matching mean and variance) lead to two equations in two unknowns (slope and intercept of the line), as follows. The desired linear equation in transformed space is

$$\hat{y}_C' = a + bx_C' \tag{3}$$

where the primes indicate transformed flows, and a and b are the unknown parameters which we wish to find. To match the mean and variance in the retransformed flows (cfs), we require:

$$\sum \hat{y}_c = \sum y_c \tag{4}$$

and

$$\sum (\hat{y}_C - m(\hat{y}_C))^2 = \sum (y_C - m(y_C))^2$$
 (5)

where summations are taken over the concurrent data. Using Eq. (4), Eq. (5) simplifies to

$$\sum \hat{y}_c^2 = \sum y_c^2 \tag{6}$$

Eq. (3) may be written in terms of retransformed flows by using Eq. (1):

$$\widehat{y}_{C} = \begin{cases} \left(\lambda_{S}(a + bx'_{C}) + 1\right)^{1/\lambda_{S}} & \text{if } \lambda_{S} \neq 0 \\ \exp(a + bx'_{C}) & \text{if } \lambda_{S} = 0 \end{cases}$$
(7)

where  $\lambda_S$  is the exponent in Eq. (1) for the short record. Eq. (7) may be substituted into Eqs. (4) and (6), leading to two equations in two unknowns, a and b, implying a unique solution for the line which meets the constraints.

Solution of the modified, hybrid MOVE. I equations (MOVE. Ia)

A closed-form solution of the equations was not apparent, and there are no general techniques for numerical root-finding in two dimensions. However, it was discovered that the equations could be solved without difficulty in nearly all cases (only one of over 100 cases proving troublesome), owing to two favorable conditions. The MOVE 1 solution provides good initial guesses for the slope and intercept, and the equations are largely decoupled, in that the variance is affected primarily by the slope, and the mean by the intercept.

Therefore, the successful numerical scheme involved alternately solving the mean-matching equation (4) for the intercept, and the variance-matching equation (6) for the slope. Using the method of successive approximations, each solution was adopted as the slope or intercept for the next calculation, to convergence. Solution of the root for each equation was performed using a polynomial interpolation function (uniroot) in the statistical computing language S-plus. (This function requires pre-bracketing the root, which was performed by stepping away from the initial guess, and reversing with smaller steps if one step led to imaginary retransformed flows. This adds complications because for some values of  $\lambda_S$  and  $x_C$ , the solution to Eq. (7) is complex so that these values must be avoided.)

Examination of the results using matching in the original space showed great improvement in estimation of high flows, but degraded estimation of low flows. This occurred because low flows have much less relative impact on the variance and mean in the original space than in transformed (e.g., logarithmic) space.

This fact, however, led to its own solution, in that it is possible to improve low-flow estimates while retaining accurate overall statistical matching. A hybrid scheme was adopted, as illustrated by hypothetical Figures 1 and 2. Figure 1 shows an example MOVE.1 curve (for which mean and variance are matched in transformed space) and a possible corresponding curve labeled MOVE.1b, based on matching in untransformed space. (The MOVE.1b slope may be lesser than that of MOVE.1, and actual curves are usually much closer.) As depicted in Figure 2, the hybrid scheme uses the MOVE.1b curve for flows above the intersection point  $Q_C$  of the curves, and MOVE.1 below. The estimated mean and variance of the concurrent flows then differ slightly from observations. Judging that matching the mean is of primary importance, the slope of the high-flow branch is then tweaked to produce exact matching of the mean, as shown to an exaggerated degree by the arrow in Figure 2. In practice, tweaking had little effect on variance, which remained much closer to observations than under MOVE.1. The resulting modified MOVE.1 method is labeled MOVE.1a in Figure 2.

#### Discussion

Each figure referred to in this discussion contains a plot showing flows in the short record for the period concurrent with the base record. Included are both the measured record and that which would have been estimated for the concurrent period by the extension curve.

Three types of plot are presented:

- flow duration curve, showing flow magnitude (log scale) vs. exceedance probability;
- hydrograph, showing flow versus date, covering the first six years of the concurrent period (chosen arbitrarily, as more years make the plots hard to read), with both log and linear axes used for flows; and
- scatterplot, plotting each short record flow (y axis) against the base record flow (x axis) on the same date. Note that the estimated flows form the extension curve.

The term "MOVE.1a" in the legend indicates the data were generated using the modified, hybrid MOVE method, while "MOVE.1" refers to the original method.

## Typical results

Figure 3 shows a flow duration curve for record 11 extended by record 43. This pair is somewhat typical in that their correlation is at the median of all extended pairs, at 0.948. The scatterplot for this pair is in Figure 4, and a hydrograph is in Figure 5. The important aspect of the hydrograph is that the frequencies of low and high flows are about right. Because flows at the short-record gage are correlated to those at the base gage, the temporal behavior of the estimated record tracks observations to a degree, but such tracking is not an explicit objective of the extension technique.

An illustration of the MOVE modifications is provided by flow duration curves for record 25, extended by 16. The full curve is in Figure 6, showing both the measured flows and those estimated by the modified MOVE method, MOVE 1a. Because this station pair is highly

correlated, with r = 0.97, one would expect a close match between the curves. Nonetheless, the MOVE.1 estimates fall above the observations, as seen in Figure 7, which shows the highest 10% of measured discharges, plus those estimated using MOVE.1 and the modified method, MOVE.1a. The improvement with MOVE.1a is clear. The improvement stems from the fact that the statistical matching is performed in terms of untransformed flow rates, which are more sensitive to high flows.

The following table compares the concurrent-period statistics of the two short records discussed above:

	Mean, cfs			Standard Deviation, cfs			
Record number	Measured	Estimated MOVE.1	Estimated MOVE.1a	Measured	Estimated MOVE.1	Estimated MOVE.1a	
11	31.1	34.1	31.1	54.9	68.9	58.6	
25	107.1	111.1	107.1	177.2	194.5	182.2	

Flow extension objectives and limitations

The modification of the extension technique raised issues concerning the objectives of the extension. The main distinction to be made is between estimation of flows on specific days, and generation of a representative data set - i.e., a flow duration curve. These are fundamentally different objectives.

When estimating a flow on a specific day, we recognize that for any specific flow rate in the base stream, the short record exhibits a range of possible flows - hence the scatter in the scatterplot. This scatter lies about a "line of means," which we commonly estimate using linear least-squares regression; points on this line represent our estimates of the mean short-record flow for each flow in the base stream. An estimate of the flow on a given day will be the point on the line corresponding to the measured base record flow, perhaps with a confidence interval reflecting the scatter about this line. In using this point estimate, we are in essence discarding the scatter as random noise about the true relationship between the streams.

However, the true relationship in fact does include this scatter. That is, this variation about the line of means is real and should be reflected in any representative data set we synthesize for the short record. Discarding this variation, as in least-squares regression, leads to a data set with insufficient variance (Hirsch, 1982). The objective of the MOVE family of techniques is to find a line which produces sufficient variance in the synthesized record; in MOVE.1 "sufficient variance" is that which produces the same variance during the concurrent period as that which is measured.

If the objective is to produce a flow duration curve, another important consideration is the type of flows of interest: high, low, mean, those producing cumulative transport, etc. There is some tradeoff in adopting one technique over another, though the modified MOVE.1 method described above greatly reduces this tradeoff, and should make it small for well-matched pairs of stations.

The caveat that the extended data sets do not predict flows on specific days points to

other limitations. Strictly, flows in the extended data sets should not be associated with specific dates, meaning they cannot be used to estimate serial correlation or statistics for time periods shorter than the full extension period. If monthly statistics are desired, the technique could be applied separately to each month. Actually, serial correlation is captured to a degree by following the base station flows; presumably, better, directed techniques could be employed, but the extended data sets might serve as suitable estimates of serial correlation or monthly statistics.

### References

- Hirsch, R.M., A comparison of four streamflow record extension techniques, *Water Resour*. Res., 18, 1081-1088, 1982.
- Moog, D.B., P.J. Whiting, and R.B. Thomas, 1999, "Streamflow record extension using power transformations and application to sediment transport," *Water Resour. Res*, 35(1), 243-254.
- Stedinger, J.R., R.M. Vogel, and E. Foufoula-Georgiou, Frequency analysis of extreme events, ch. 18 in *Handbook of Hydrology*, D.R. Maidment, ed., McGraw-Hill, New York, 1993.

Appendix. Record and gage numbers, base gages, and record periods for the San Juan National Forest.

# Bold indicates base gage.

Italics indicate station not recommended for base gage extension. # = base gage or best base gage candidate.

Record Number	Gage Number	Gage Name	Base Record Number	Correl. to Base Record	Concurr. per. Extended per.
1	9165000	Dolores River Below Rico, Co.	43	0.97	1951-97 1912-98
2	9166000	West (Fork) Dolores River	3	0.97	1941-44 1921-98
3	9166500	Dolores River At Dolores	-	-	1921-98
4	9166950	Lost Canyon Ck Nr Dolores	44	0.50	1984-98
5	9167000	Lost Canyon Ck At Dolores	44	0.65	1941-48
6	9167450	Plateau Creek Nr Mouth, Near	44	0.71	1982-83 1917-98
7	9167500	Dolores River Nr Mcphee, Co.	16	0.79	1938-52 1910-14,35-98
<b>8</b>	9339900	Ef San Juan R Ab Sand Ck, Nr	16	0.97	1935-96 1910-14,35-98
9	9340000	East Fork San Juan R Nr Pag	16	0.98	1956-80 1910-14,35-98
10	9340500	Wf San Juan R Ab Borns Lk	43	0.96	1937-53 1912-98
11	9341200	Wolf Creek Nr Pagosa Springs	43	0.95	1968-75 1912-98
, 12	9341300	Wolf Cr At Wolf Cr Campgr	43	0.96	1984-87, 97-98 1912-98
13	9341350	Windy Pass Cr Nr Pagosa Spr	44	0.78	1984-87

14	9341500	West Fork San Juan R Nr Pag	16	0.98	1935-60, 84-98
15	9342000	Turkey Ck Nr Pagosa Spring	16	0.89	1937-49 1910-14,35-98
16	9342500	San Juan R At Pagosa Spr	-	-	1910-14,35-98
17	9343000	Rio Blanco Nr Pagosa Springs	16	0.97	1935-71 1910-14,35-98
18	9343300	Rio Blanco Bl Blanc Div Dam	16	0.74	1971-98 1910-14,35-98
19	9343500	Rito Blanco Nr Pagosa Springs	16	0.86	1935-52 1910-14,35-98
20	9344300	Navajo River Above Chromo	16	0.97	1956-70 1910-14,35-98
21	9345500	Little Navajo R At Chromo	16	0.65	1935-52
22	9346000	Navajo River At Edith, Co.	16	0.97	1912-14,35-70 1910-14,35-98
23	9347200	Middle Fork Piedra R Nr Pag	43	0.95	1969-75 1912-98
24	9347205	Middle Fork Piedra River Near	16	0.98	1977-83 1910-14,35-98
25	9347500	Piedra R At Bridge Rngr Sta	16	0.97	1936-41, 46-54 1910-14,35-98
26	9348500	Williams C Nr Bridge Ra Sta	16	0.96	1937-41,46-49 1910-14,35-98
27	9349000	Weminuche C Nr Brdge Ra Sta	16	0.96	1937-41,46-69 1910-14,35-98
28	9349500	Piedra River Near Piedra, Co.	16	0.98	1939-73 1910-14,35-98
29	9352900	Vallecito Creek Nr Bayfield	-	-	1962-98
30	9353500	Los Pinos River Near Bayfield	43	0.97	1928-40 1912-98
31	9358000	Animas River At Silverton, Co.	29	0.93	1991-98 1962-98

32	9358550	Cement Creek At Silverton, Co	43	0.94	1991-98 1912-98
33	9358900	Mineral Creek Above Silverton	29	0.95	1968-75 1962-98
34	9359000	Mineral Creek Near Silverton	43	0.93	1936-49 1912-98
35	9359010	Mineral Creek At Silverton, Co	29	0.95	1991-98 1962-98
36	9359020	Animas River Below Silverton	29	0.95	1991-98 1962-98
37	9359100	Lime Creek Near Silverton	43	0.90	1956-61 1912-98
38	9359500	Animas River Above Tacoma	43	0.98	1945-56 1912-98
39	9361000	Hermosa Creek Near Hermosa	16	0.95	1940-80 10-14,20-28,35-98
40	9361200	Falls Creek Near Durango, Co.	16	0.88	1959-65 1910-14,35-98
41	9361400	Junction Creek Near Durango	44	0.94	1959-65 1917-98
42	9362000	Lightner Creek Near Durango	44	0.82	1927-49 1917-98
43	9361500	Animas River At Durango	-	-	1912-98
44	9365500	La Plata River At Hesperus	-	-	1917-98
45	9368500	West Mancos R Nr Mancos	3	0.88	1938-53 1921-98
46	9369000	East Mancos R Near Mancos,	44	0.92	1937-51
47	9369500	Middle Mancos R Nr Mancos	44	0.90	1938-51
48	9370000	Mancos River Near Mancos	3	0.95	1931-38 1921-98
49	9357500	Animas River At Howardsville	29	0.94	1962-82 1962-98

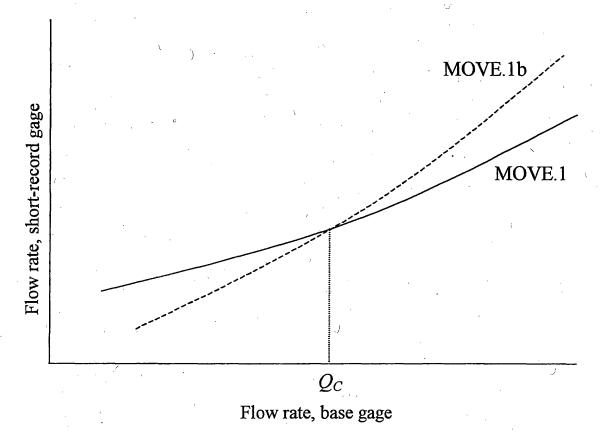


Figure 1 Hypothetical diagram comparing curves of MOVE.1 and MOVE.1b (mean and variance matched in untransformed space). Their intersection occurs at a base gage flow of  $Q_C$ . The relative slopes shown are arbitrary, and the actual difference is generally smaller.

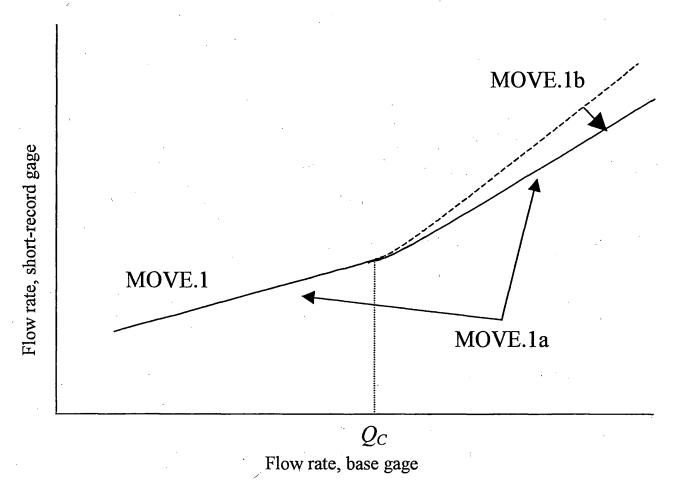


Figure 2 Hypothetical diagram showing derivation of modified MOVE method, MOVE.1a. The lower branch of MOVE.1 and the upper branch of MOVE.1b form the MOVE.1a curve, after tweaking the upper branch slope (shown here to an exaggerated degree). The branches are divided at the crossover point,  $Q_C$ .

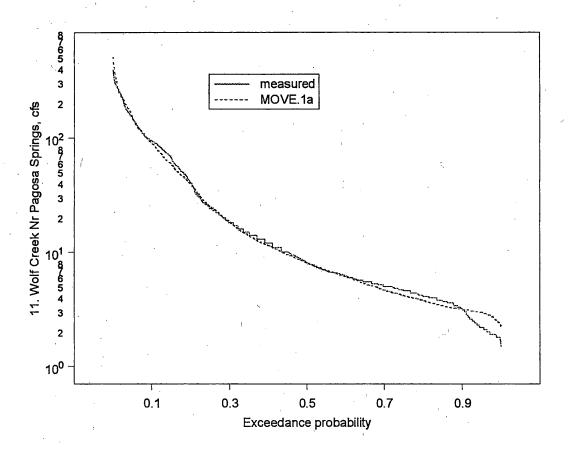


Figure 3 Flow duration curves, measured and estimated via modified MOVE.1, for the concurrent period of record 11. The concurrent period consists of dates with flows in both record 11 and its base station, record 43. Record 11 is typical of those extended in that its base station correlation lies at their median.

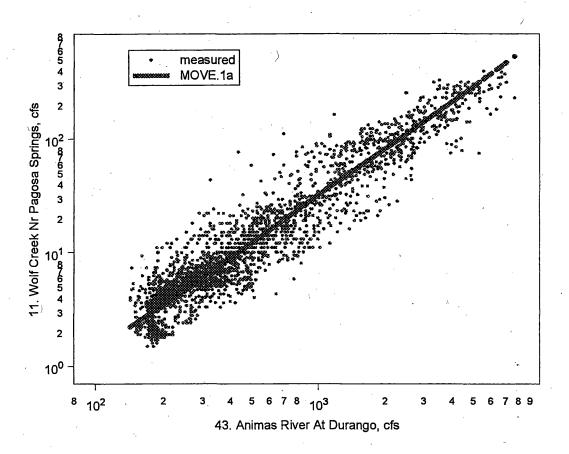


Figure 4 Measured flow rates in record 11 vs. flow on the same date in record 43, with the modified MOVE.1 line (MOVE.1a), indicating the estimated flow corresponding to each base station flow.

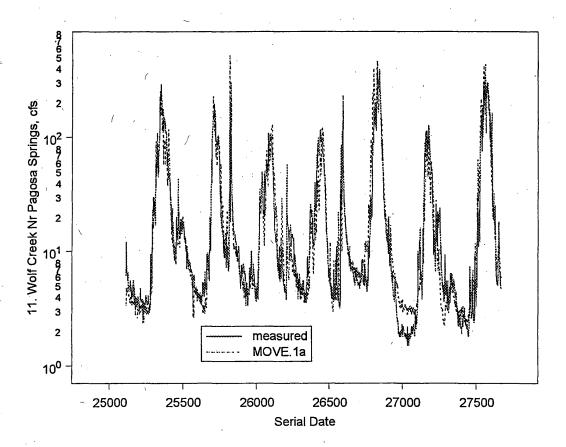


Figure 5 Measured and estimated hydrograph for record 11, water years 1969-1975. Variance is very close.

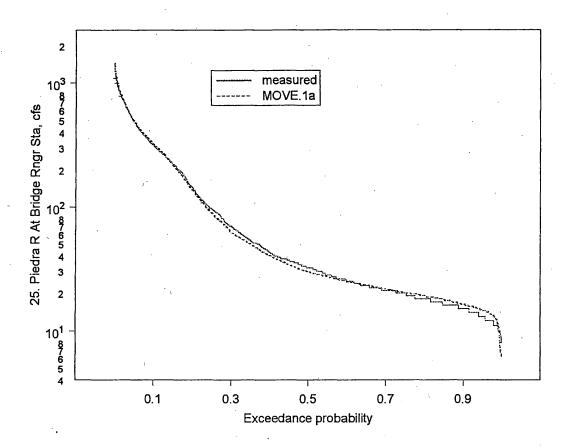


Figure 6 Measured and estimated flow curves for record 25, which has a high correlation to its base stream, 16.

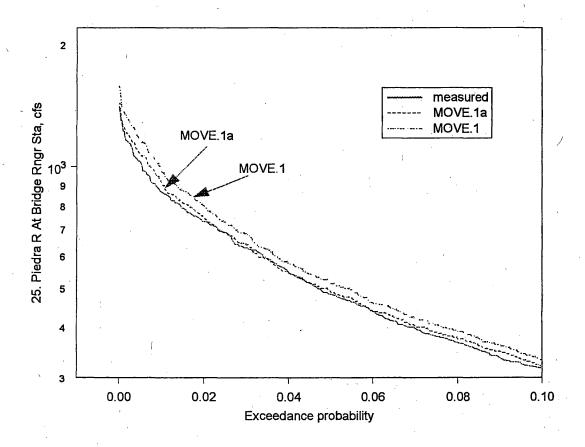


Figure 7 Flow duration curves for the highest 10% of flows in record 25, showing the high-flow error associated with MOVE 1 even in a well-correlated pair, and its improvement using the modified method, MOVE 1a.